ARRL Periodicals Archive – Search Results
A membership benefit of ARRL and the ARRL Technical Information Service

ARRL Members: You may print a copy for personal use. Any other use of the information requires permission (see Copyright/Reprint Notice below).

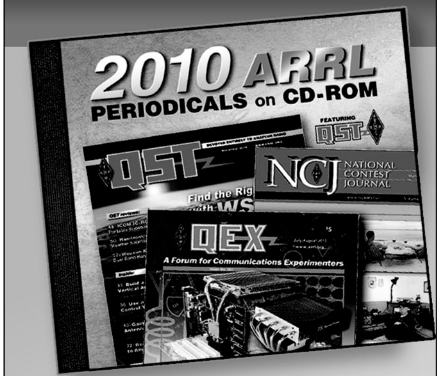
Need a higher quality reprint or scan? Some of the scans contained within the periodical archive were produced with older imaging technology. If you require a higher quality reprint or scan, please contact the ARRL Technical Information Service for assistance. Photocopies are $3 for ARRL members, $5 for nonmembers. For members, TIS can send the photocopies immediately and include an invoice. Nonmembers must prepay. Details are available at www.arrl.org/tis or email photocopy@arrl.org.

QST on CD-ROM: Annual CD-ROMs are available for recent publication years. For details and ordering information, visit www.arrl.org/qst.

Non-Members: Get access to the ARRL Periodicals Archive when you join ARRL today at www.arrl.org/join. For a complete list of membership benefits, visit www.arrl.org/benefits.

QEX Issue: Apr 1994
Title: Phase-Shift Network Analysis and Optimization
Author: Kevin Schmidt, W9CF

Click Here to Report a Problem with this File



Phase-Shift Network Analysis and Optimization

Analyzing a phase-shift network shows how its performance varies with component tolerances.

by Kevin Schmidt, W9CF

Introduction

The phasing method of single-sideband generation or detection requires two signals with a 90° relative phase shift over the audio frequency range. The phasing method has never been very popular, particularly once relatively inexpensive filters became available. In the future, presumably, digital signal-processing techniques will perform the necessary audio phase shifting or directly generate the radio frequency single-sideband signal. Why then should you be interested in audio phase-shift networks? Perhaps because they are relatively low cost, easy to build and are fun to play with. In addition, the techniques that I describe here are useful for efficient analysis of other cascaded networks.

For many years, the *ARRL Handbook* has included a circuit for an audio phase-shift network designed by HA5WH.[1] I have not located the original reference for this network. The *Handbook* claims that the circuit gives approximately 60 dB of opposite sideband suppression using 10% tolerance components. This flies in the face of the usual result that you need 1% components to get around 40-dB suppression. In this article, I will analyze and give design equations for this type of network. Unfortunately, this analysis shows that using 10% tolerance components *can* lead to poor sideband suppression. With ideal components the network can give excellent performance, and by using either high-tolerance components or well-matched lower-tolerance components, the network still can give good performance.

Notes appear on page 23.

6510 S. Roosevelt St
Tempe, AZ 85283
email:kevin.schmidt@asu.edu (Internet)

In the following sections I give the general formula for the sideband suppression in terms of the phase and amplitude errors in the phasing network; derive an efficient method of analyzing a general network of the HA5WH type; give the analysis of an ideal realization of the network; describe the optimization of the network in terms of easily calculated elliptic functions; and, give the effects of component tolerances. The result is a set of simple design equations for the ideal network and an estimate of the sensitivity to component tolerances. A set of FORTRAN programs that implement the methods described are available for downloading.

The Effects of Phasing Errors on Sideband Suppression

The phasing method generates a single-sideband signal, given mathematically as $\cos((\omega_c \pm \omega_a)t)$, where the + (or −) sign gives the upper (or lower) sideband, and $\omega_c = 2\pi f_c$ where f_c is the carrier frequency. Similarly, $\omega_a = 2\pi f_a$ where f_a is the audio modulating frequency. The cosine can be written as

$$\cos\left((\omega_c \pm \omega_a)t\right) = \cos\left(\omega_c t\right)\cos\left(\omega_a t\right) \mp \sin\left(\omega_c t\right)\sin\left(\omega_a t\right) \quad \text{Eq 1}$$

the basic equation of the phasing method. The multiplications on the right-hand side are accomplished using balanced modulators, and the two audio frequencies (as well as the two radio frequencies) must be 90° out of phase and

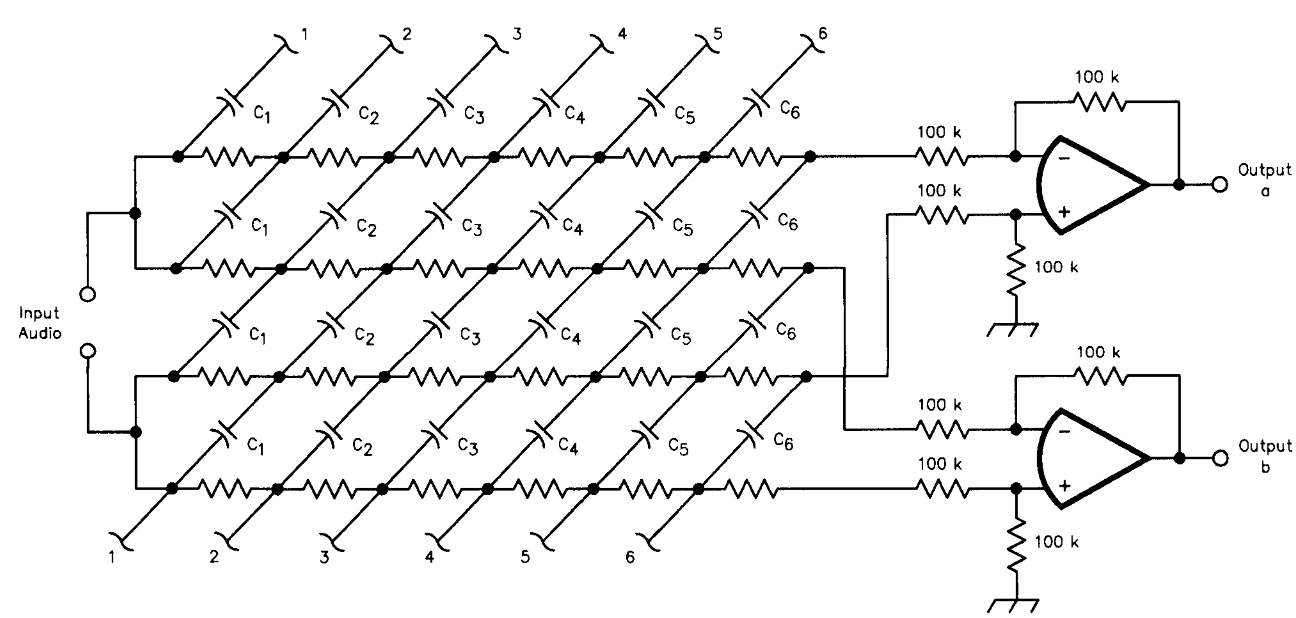


Fig 1—The schematic diagram of the HA5WH wide-band phase-shift network.

of equal amplitude. I will assume that the radio frequencies are exactly 90° out of phase, and of equal amplitude. Using the usual complex notation with $V_A e^{j\omega_a t}$ to be one audio signal, and $V_B e^{j\omega_a t}$ to be the other, the result of using a nonideal phasing network will be

$$\mathrm{Re}\left[\cos(\omega_c t)V_A e^{j\omega_a t} + \sin(\omega_c t)V_B e^{j\omega_a t}\right] = \frac{1}{2}\mathrm{Re}\left[e^{j(\omega_c+\omega_a)t}(V_A - jV_B) + e^{-j(\omega_c-\omega_a)t}(V_A + jV_B)\right] \quad \text{Eq 2}$$

and the sideband suppression (or enhancement) is given by

$$20\log_{10}\left|\frac{V_A + jV_B}{V_A - jV_B}\right| \quad \text{Eq 3}$$

Notice if $|V_A|$ equals $|V_B|$, that is if the two signals have equal amplitude then for a phase error of δ, the suppression in dB is simply,

$$-20\log_{10}\left|\tan\left(\frac{\delta}{2}\right)\right| \quad \text{Eq 4}$$

Analyzing the HA5WH Network

Fig 1 gives the circuit diagram of the HA5WH network as shown in the *ARRL Handbook*. Given this circuit, it is easy to analyze the network numerically using a mesh or nodal analysis. The disadvantage of this brute force approach is that it gives no insight into why the network works, or how changes in the network affect its performance. I will therefore describe a method that is both more efficient numerically, and, by using the symmetry of the ideal network, leads to simple design equations.

The network consists of six sections each with four input connections and four output connections. One of these sections is shown in Fig 2. I have labeled the input voltages and currents V_1, V_2, V_3, V_4, I_1, I_2, I_3, I_4. The corresponding output voltages and currents are labeled V'_1, V'_2, V'_3, V'_4, I'_1, I'_2, I'_3, I'_4. A straightforward nodal analysis of this network gives the eight linear equations represented by the matrix equation

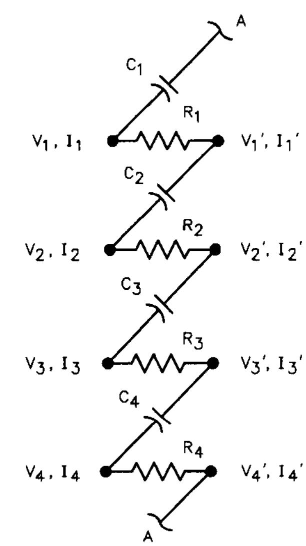


Fig 2—The schematic diagram of one section of an HA5WH network.

$$\begin{pmatrix} I \\ I' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V \\ V' \end{pmatrix} \quad \text{Eq 5}$$

where V, V', I, I' are length 4 vectors, and the M_{ij} are 4-by-4 matrices. Eq 5 compactly represents the eight equations that are the requirements of current conservation at each of the nodes of the network section. The M_{ij} matrices are

$$M_{11} = \begin{pmatrix} \frac{1}{R_1} + j\omega C_1 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} + j\omega C_2 & 0 & 0 \\ 0 & 0 & \frac{1}{R_3} + j\omega C_3 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} + j\omega C_4 \end{pmatrix}$$

$$M_{12} + \begin{pmatrix} -\frac{1}{R_1} & 0 & 0 & -j\omega C_1 \\ -j\omega C_2 & -\frac{1}{R_2} & 0 & 0 \\ 0 & -j\omega C_3 & -\frac{1}{R_3} & 0 \\ 0 & 0 & -j\omega C_4 & -\frac{1}{R_4} \end{pmatrix}$$

$$M_{21} = \begin{pmatrix} \frac{1}{R_1} & j\omega C_2 & 0 & 0 \\ 0 & \frac{1}{R_2} & j\omega C_3 & 0 \\ 0 & 0 & \frac{1}{R_3} & j\omega C_4 \\ j\omega C_1 & 0 & 0 & \frac{1}{R_4} \end{pmatrix}$$

$$M_{22} = \begin{pmatrix} -\frac{1}{R_1} - j\omega C_2 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_2} - j\omega C_3 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_3} - j\omega C_4 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_4} - j\omega C_1 \end{pmatrix}$$

Eq 6

In exact analogy with cascading two-port networks using *ABCD* matrices, to cascade these network sections I define a new matrix equation,

$$\begin{pmatrix} V' \\ I' \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} V \\ I \end{pmatrix} \qquad \text{Eq 7}$$

Solving for the A_{ij} matrices gives,

$$\begin{aligned} A_{11} &= -M_{12}^{-1} M_{11} \\ A_{12} &= M_{12}^{-1} \\ A_{21} &= M_{21} - M_{22} M_{12}^{-1} M_{11} \\ A_{22} &= M_{22} M_{12}^{-1} \end{aligned} \qquad \text{Eq 8}$$

where M_{12}^{-1} is the inverse of the matrix M_{12}.

Labeling the 8-by-8 matrices for each of the n sections of the network by $A^{(1)}, A^{(2)}, \ldots A^{(n)}$, the matrix relating the input to the output of the full network is $\tilde{A}$ made up of the four 4-by-4 matrices $\tilde{A}_{ij}$,

$$\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix} = \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{pmatrix} \begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} \qquad \text{Eq 9}$$

where $\tilde{A}$ is the matrix product $A^{(1)}A^{(2)}A^{(3)} \ldots A^{(n)}$.

The *Handbook* circuit drives four resistors on the four output connections. Labeling these as $R_1^{(out)} R_2^{(out)} R_3^{(out)}$, $R_4^{(out)}$, and defining a 4-by-4 load matrix L,

$$L = \begin{pmatrix} \frac{1}{R_1^{(out)}} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2^{(out)}} & 0 & 0 \\ 0 & 0 & \frac{1}{R_3^{(out)}} & 0 \\ 0 & 0 & 0 & \frac{1}{R_4^{(out)}} \end{pmatrix} \qquad \text{Eq 10}$$

I can write the relationship between the output voltage and current as,

$$(I_{out}) = L(V_{out}) \qquad \text{Eq 11}$$

Solving for I_{out} and back substituting gives the final network matrix equation relating the four output voltages to the four input voltages,

$$(V_{out}) = \left(1 - \tilde{A}_{12}\tilde{A}_{22}^{-1}L\right)^{-1}\left(\tilde{A}_{11} - \tilde{A}_{12}\tilde{A}_{22}^{-1}\tilde{A}_{21}\right)(V_{in}) \qquad \text{Eq 12}$$

where 1 in Eq 12 stands for the unit matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \text{Eq 13}$$

If the output resistors are large compared to the other circuit impedances, L can be taken to be zero. In that case the equations simplify to,

$$(V_{out}) = \left(\tilde{A}_{11} - \tilde{A}_{12}\tilde{A}_{22}^{-1}\tilde{A}_{21}\right)(V_{in}) \qquad \text{Eq 14}$$

The *Handbook* network has (V_{in}) proportional to

$$(V_{in}) \propto \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \qquad \text{Eq 15}$$

After calculating the $\tilde{A}$ and L matrices from the circuit values, the output signals need to be combined as,

$$\begin{aligned} V_{out,1} - V_{out,3} &= V_A \\ V_{out,2} - V_{out,4} &= V_B \end{aligned} \qquad \text{Eq 16}$$

and the sideband suppression is given by Eq 3. The relative amplitude and phase of the signals can also be calculated. Most phase-shift networks are based on all-pass networks so that the amplitude of all signals is equally attenuated. The HA5WH network is not an all-pass network. Ideally, we want both good sideband suppression and we want V_A and V_B to be constant in amplitude and phase across the passband of the audio circuit.

I have written a FORTRAN program to implement the analysis of this section. It is available for download from the ARRL BBS (203 666-1578) and via Internet FTP from ftp.cs.buffalo.edu, in the \pub\ham-radio directory. The file name is pshift.zip. If the matrices that are inverted become singular, the above analysis breaks down at the singular points. For example, M_{12} becomes singular when

$$\omega^4 = \frac{1}{R_1R_2R_3R_4C_1C_2C_3C_4} \quad \text{Eq 17}$$

Near these points, roundoff error in the calculations will be large. For the analysis done here, this is not a big problem. However, analysis on networks with many sections or near singular points will require more numerical care than I have taken in the FORTRAN program, or the use of a standard formulation where the full set of network equations are solved at once.

Analysis of the Ideal Cyclic Network

The design of the HA5WH network, as shown in the *Handbook*, has four identical resistors and four identical capacitors in each of the six network sections. This means the network is invariant under a cyclic interchange of the ordering of its ports. That is, if we were to relabel the ports by letting 1 become 2, 2 become 3, 3 become 4, and 4 become 1, we would obtain exactly the same equations describing the network. Such invariances are treated generally using the mathematics of group theory, which greatly simplifies the study of the system with symmetries.[2] The ideal HA5WH network has what is known as cyclic 4 or C_4 symmetry. The network equations can be analyzed using group theory. Analysis of the character of the matrix that represents the cyclic operator shows that each of the four irreducible representation of C_4 appears once. These therefore correspond to the four eigenvectors of the $\tilde{A}$ matrices, which can then be written down immediately.

Most hams probably are unfamiliar with group theory, however, the results can be easily verified without using group theory. The right eigenvectors $\psi^{(m)}$ and the eigenvalues λ_m of a matrix M are defined by finding the solutions to the equations,

$$M\psi^{(m)} = \lambda_m \psi^{(m)} \quad \text{Eq 18}$$

That is, multiplying the eigenvector by the matrix gives the same eigenvector back as the result, simply multiplied by the eigenvalue. The effect of multiplying a matrix times one of its eigenvectors is to simply multiply the eigenvector by the eigenvalue.

The cyclic eigenvectors in our basis are those that change by a constant phase between the elements, with the same phase change between the last and first elements. This gives,

$$\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\j\\-1\\-j\end{pmatrix}, \begin{pmatrix}1\\-j\\-1\\j\end{pmatrix} \quad \text{Eq 19}$$

By direct matrix multiplication, it is easily verified that these are the eigenvectors of *all* the M matrices if all the R and C values are the same in a network section. This is a direct consequence of the cyclic 4 symmetry. Further, since the $\tilde{A}$ matrices are combinations of products of the M matrices, these same vectors are the eigenvectors of the $\tilde{A}$ matrices. Since V_{out} is given as a combination of $\tilde{A}$ matrices times V_{in}, if we express V_{in} as a linear combination of the four eigenvectors, V_{out} will be given by taking this same linear combination and multiplying each term by an appropriate eigenvalue. The network must then be designed to produce a 90° relative phase shift.

The input to the HA5WH network contains only the last two eigenvectors written above. That is

$$V_{in} = \begin{pmatrix}1\\1\\-1\\-1\end{pmatrix} = \frac{1-j}{2}\begin{pmatrix}1\\j\\-1\\-j\end{pmatrix} + \frac{1+j}{2}\begin{pmatrix}1\\j\\-1\\-j\end{pmatrix} \equiv \frac{1-j}{2}\psi_a + \frac{1+j}{2}\psi_b \quad \text{Eq 20}$$

where the last line defines the relevant eigenvectors as ψ_a and ψ_b. Further, the output is also not sensitive to the first two eigenvectors if the output impedances are identical and the operational amplifiers have good common-mode rejection. Having both of the conditions will be helpful if the cyclic symmetry is broken because of component tolerances.

With the input as in Eq 20, the output will in general be

$$V_{out} = \frac{1-j}{2} g_a \psi_a + \frac{1+j}{2} g_b \psi_b \quad \text{Eq 21}$$

and the two outputs to the balanced modulators will be

$$\begin{aligned} V_A &= (1-j)g_a + (1+j)g_b \\ V_B &= (1-j)jg_a - (1+j)jg_b \end{aligned} \quad \text{Eq 22}$$

the suppression in dB is found using Eq 3,

$$20\log_{10}\left|\frac{g_a}{g_b}\right| \quad \text{Eq 23}$$

So to design a good network, we must eliminate one of these final two eigenvectors.

The analysis so far shows how the HA5WH network can be motivated. The C_4 eigenvectors have equal amplitudes for the four voltages and have phase shifts between adjacent ports of 0°, +90°, 180° and 270°. This last is equivalent to a phase shift of –90°. We want to choose the network drive, connections and component values to select one of the two 90° phase-shifted eigenvectors. As an aside, the same ideas could be used to design a 60° relative phase shift by using a network invariant under the group C_6, or a 45° shift from C_8, etc.

The first step in selecting the component values is to calculate the eigenvalues of the four M matrices. By direct multiplication, I get

$$\begin{aligned} &\lambda^a_{11} = \lambda^b_{11} = \frac{1}{R} + j\omega C \\ &\lambda^a_{12} = -\frac{1}{R} - \omega C, \lambda^b_{12} = -\frac{1}{R} + \omega C \\ &\lambda^a_{21} = \frac{1}{R} - \omega C, \lambda^b_{21} = \frac{1}{R} + \omega C \\ &\lambda^a_{22} = \lambda^b_{22} = -\frac{1}{R} - j\omega C \end{aligned} \quad \text{Eq 24}$$

where the superscript a or b indicates the eigenvalue corresponds to the eigenvector ψ_a or ψ_b, respectively.

The effect of one of the A matrices, when a single eigenvector is input, is given by replacing the M matrices in

Eq 8 by their eigenvalues. After a little algebra, I get,

$$A^a = \frac{1}{1+\omega RC}\begin{pmatrix} 1+j\omega RC & -R \\ -2j\omega C & 1+j\omega RC \end{pmatrix}$$

$$A^b = \frac{1}{1-\omega RC}\begin{pmatrix} 1+j\omega RC & -R \\ -2j\omega C & 1+j\omega RC \end{pmatrix} \quad \text{Eq 25}$$

The A^b matrix is proportional to A^a. If we feed the section of the network with a linear combination of ψ_a and ψ_b, the section suppresses ψ_a relative to ψ_b by a factor of

$$\frac{1-\omega RC}{1+\omega RC} \quad \text{Eq 26}$$

The HA5WH network has the properties that the magnitude of the ratio given in Eq 26 is always less than one for positive frequencies and is exactly zero for $\omega=1/(RC)$. The first property says that additional network sections can only improve the relative 90° phase shift of the outputs. The second says that we can set the frequencies of exact 90° phase shift by selecting the R-C values of single network sections. These two properties greatly simplify the design and optimization of the network.

The sideband suppression at a single frequency is given for an n section network, with R-C values in section i given by R_i and C_i, as

$$\text{Suppression} = 20\sum_{i=1}^{n} \log_{10}\left|\frac{1-\omega R_i C_i}{1+\omega R_i C_i}\right| \quad \text{Eq 27}$$

A simple method of picking the R-C values for each section is to use a computer to plot the above result, and adjust n and R_iC_i to achieve the required suppression. This is, in fact, the obvious technique to use if you are trying to design with a set of parts already in your junk box. However, the form of the suppression makes it easy to select optimum values, as seen in the next section.

Optimizing the Sideband Suppression

The optimum values of R_iC_i can be easily calculated using elliptic functions. Typically, we want the worst-case suppression to be the highest possible. This leads us to the equal ripple or Chebychev approximation. The mathematics are straightforward and given in detail by Saraga.[3] For an upper and lower frequency of f_u and f_l respectively, the R_iC_i values for an n-section network are,

$$R_iC_i = \frac{\mathrm{dn}\left(\frac{2i-1}{2n}\mathrm{K}(k),k\right)}{2\pi f_l} \quad \text{Eq 28}$$

where

$k = \sqrt{1-(f_l/f_u)^2}$, $\mathrm{K}(k)$

is the complete elliptic integral of the first kind, and $\mathrm{dn}(u,k)$ is a Jacobi elliptic function.[4,5]

One of the FORTRAN programs calculates the R_iC_i values given the upper and lower frequencies and the n value. In Table 1, I give some calculated values for some networks of interest to hams, and their theoretical sideband suppression. These theoretical results will, of course, be best cases assuming perfect components.

In passing, I note that Saraga's Taylor approximation is given by simply choosing all the R_iC_i values to be the same and equal to

$$\frac{1}{2\pi\sqrt{f_u f_l}}$$

(see note 3). Also, if maximum suppression is needed at a particular frequency (for example if you wanted to use audio tones in a single-sideband transmitter to produce frequency shift keying), it is simple to select R_iC_i values appropriate for these frequencies and then optimize the other network sections.

Effects of Amplitude Variations and Component Tolerances

So far, I have only looked at the relative phase shift of the two outputs. To have a high-quality audio signal, the network must have a flat amplitude output. Usually, this is handled by constructing, respectively, an all-pass network. Since the HA5WH network is not an all-pass form, we must examine its attenuation as a function of frequency. In Figs 3, 4, and 5, I have plotted the sideband suppression and the amplitude and phase variations of one of the output signals, for the optimal 4, 6, and 8-section filters designed for the frequency range 300 to 3000 Hz with equal value resistors. The network sections are ordered from largest RC value to smallest, as in the original HA5WH design. As shown, the amplitude variations are less than ±1 dB, the phase variation is smooth, and the sideband suppression is of the equal ripple form—as expected.

One of the main selling points given in the *Handbook* description of this network is the claim that low-tolerance components can be used to obtain a high-performance network. From the analysis in the previous section, if cyclic symmetry is maintained, the network will perform

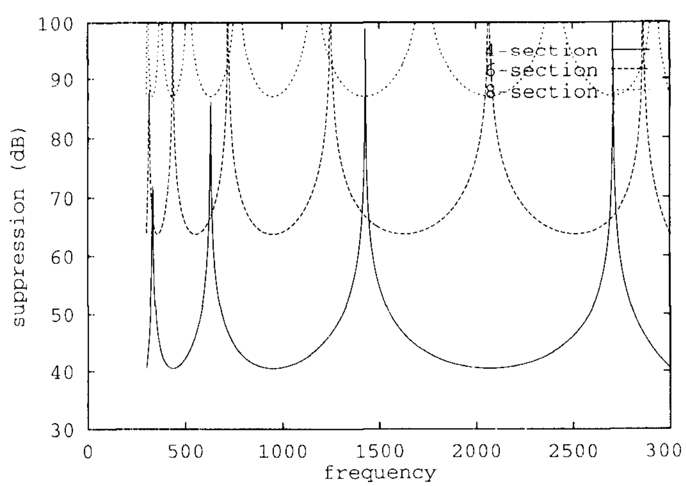


Fig 3—Ratio of the magnitude of the unwanted to wanted sideband for the 4, 6, and 8-section optimal Chebychev networks for the frequency range 300 to 3000 Hz.

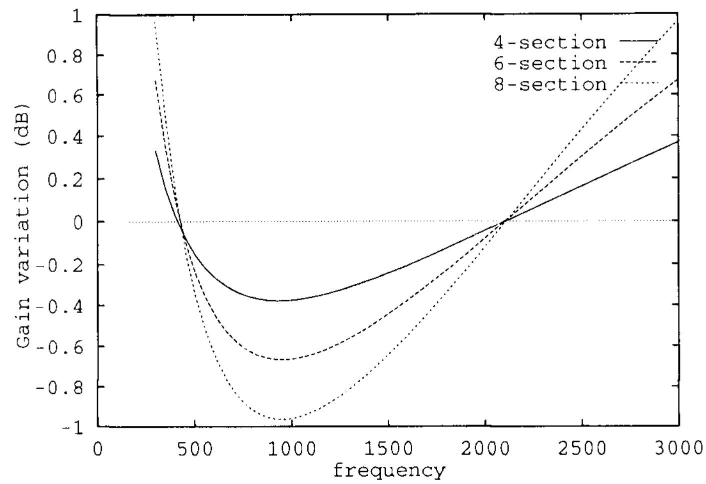


Fig 4—Relative amplitude of one output signal for the 4, 6 and 8-section optimal Chebychev networks for the frequency range 300 to 3000 Hz.

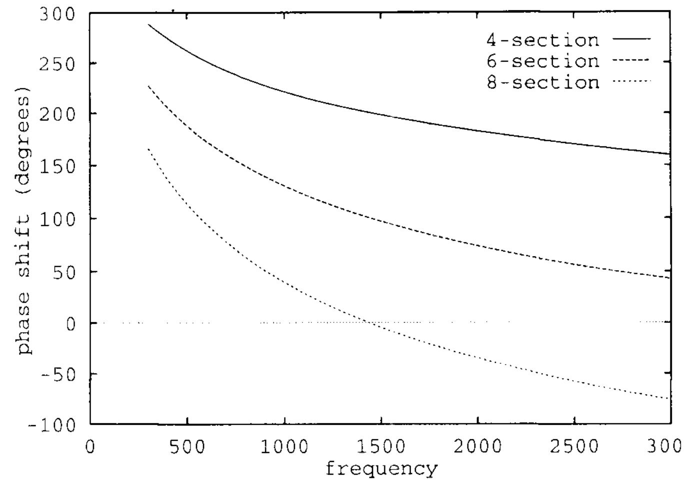


Fig 5—The phase shift of one output signal for the 4, 6, and 8-section optimal Chebychev networks for the frequency range 300 to 3000 Hz.

perfectly at the n selected frequencies corresponding to $f=1/(2\pi RC)$ for each network section. Since matching components is generally easier than measuring their values accurately, I examine the effect of a change of these node frequencies caused by perfectly matched, but low-tolerance components. Since both the resistors and capacitors can vary, using 10% components can vary the nodal frequency values by approximately 20% if both components change value in the same direction. A worst-case condition would be for all the sections to have too high or too low of a frequency by 20%. This simply shifts the network center frequency by 20%. For the optimal 6-section filter from 300 to 3000 Hz, this changes the sideband suppression from over 60 dB to about 42 dB. If a 10% variation of network node frequencies is assumed, that is 55 components, and again all the frequency changes are assumed to be in the same direction, the suppression is nearly 50 dB. This shows that relatively low-tolerance but *well-matched* components can give excellent results. Eq 27 can be used to predict the effect of changing the R-C values of each filter section due to component tolerances when the components are perfectly matched in each section. The case where unmatched R and C values in each section are used is of course the one with the most practical interest. Here, we can get an idea of what the worst-case possibilities are by looking at the cross terms between ψ_a and ψ_b when the M matrices are no longer cyclic. Typical terms give contributions like,

$$\left(\frac{1}{R_1}+\frac{1}{R_3}\right)-\left(\frac{1}{R_2}+\frac{1}{R_4}\right) \qquad \text{Eq 29}$$

or

$$\omega(C_1+C_3)-\omega(C_2+C_4) \qquad \text{Eq 30}$$

where here the subscripts 1, 2, 3 and 4 indicate the position in the network section as in Fig 2. This indicates that a single section with a tolerance t (t = 0.1 would be 10% tolerance) can reduce the overall suppression to roughly $20\log_{10}(t)$ dB. That is, 10% components could give suppressions as low as 20 dB, and 1% components as low as 40 dB if the components in a network section are not matched. Notice that to be sure to obtain 60-dB opposite sideband attenuation, components with short- and long-term tolerances of 0.1% would need to be used.

As a concrete example of this sensitivity to unmatched components, I calculated the suppression of the original HA5WH 6-section filter for the case where only the resistors in the last section have been changed by 10%. R_1 and R_3 have been raised by 10%, and R_2 and R_4 have been lowered by 10%. For ideal components, the suppression is greater than 57 dB, but changing just the resistors in the last section reduces the suppression to 26 dB, in rough agreement with the simple calculation above. Using these results to try to cook up a near worst case, I tried changing all the resistors in exactly the same manner in each section. In addition I changed all the capacitors by raising the C_2 and C_4 values by 10% and lowering the C_1 and C_3 values by 10%. The result was to further lower the unwanted sideband suppression to about 17 dB. Clearly, 10% components and bad luck will produce unacceptable sideband suppression.

One last comment on the *Handbook* circuit is the design of the operational amplifier circuit for the output. One section of this circuit is shown in Fig 6. All the resistors have the same value in the *Handbook* circuit. This does not give a balanced output and would be another source of phasing errors. If I assume that the operational amplifier input impedances are very large, the input impedance at point 2 is clearly $2R_2$. The voltage at the noninverting input is therefore $V_2/2$. The current drawn from input 1 is therefore

$$\frac{V_1 - V_2/2}{R_1}$$

and since $V_1 = -V_2$ with perfect phasing, the impedance seen at input 1 is $1.5R_1$. So $2R_2$ should be equal to $1.5R_1$, and in addition, dc balancing of the operational amplifiers may be required to compensate for input bias current. For the *Handbook* circuit, the unbalanced output resistance

Table 1

The optimal Chebychev values for some ideal HA5WH type phasing networks. f_t and f_u are the upper and lower frequencies, n is the order of the network, and f_i, where i is 1 through n, are the frequencies of exact 90° phase shift. The corresponding R-C values are $1/(2\pi f_i)$. Sup is the minimum sideband suppression over the network range in dB.

f_t	f_u	n	*Sup(dB)*	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
300	3000	4	40.5	332.2	629.8	1429.0	2709.0				
300	3000	5	52.1	320.5	500.7	948.7	1797.6	2808.1			
300	3000	6	63.7	314.2	435.5	720.3	1249.5	2066.8	2864.5		
300	3000	7	75.4	310.4	397.8	595.3	948.7	1511.8	2262.4	2899.4	
300	3000	8	87.0	308.0	374.0	519.4	771.2	1167.0	1732.7	2406.2	2922.5
200	4000	5	42.9	219.5	398.4	894.4	2008.1	3645.0			
200	4000	6	52.7	213.5	332.1	633.1	1263.6	2408.9	3747.8		
200	4000	7	62.5	209.9	294.6	497.5	894.4	1608.2	2715.5	3812.0	
200	4000	8	72.2	207.5	271.2	417.8	689.9	1159.6	1915.0	2949.6	3854.8
150	6000	6	44.7	163.6	287.7	628.9	1431.1	3128.3	5500.9		
150	6000	7	53.1	160.0	247.7	471.0	948.7	1910.7	3633.0	5626.4	
150	6000	8	61.5	157.6	223.1	381.3	696.7	1291.9	2360.2	4033.2	5710.4

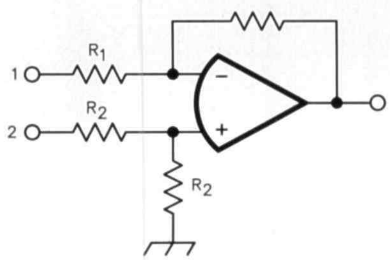


Fig 6—The schematic diagram of one operational amplifier section.

reduces the sideband suppression even in the ideal component case to about 35 dB.

Conclusion

The HA5WH network takes advantage of cyclic symmetry to give simple design equations and excellent sideband suppression with ideal components. If the cyclic symmetry is maintained, the network is not very sensitive to component tolerances. This means that the components in each of the network sections should be carefully matched. Breaking the cyclic symmetry by using unmatched components can drastically affect the performance of the network.

I have given a set of formulas and FORTRAN programs to design the optimum ideal networks and to analyze both the ideal and nonideal cases. Analyses other than the cases that I have described here can be easily done with these methods and programs.

Notes

[1] *The ARRL Handbook for the Radio Amateur*, (American Radio Relay League, Newington, 1993), and many previous editions.

[2] See for example, Cracknell, A.P., *Applied Group Theory*, (Pergaman, Oxford, 1968) for an introduction to group theory, with reprints of selected original papers.

[3] Saraga, W., "The Design of Wide-Band Phase Splitting Networks," *Proc IRE*, Vol 38, p 754 (1950).

[4] Cayley, A., *An Elementary Treatise on Elliptic Functions*, (Dover, New York, 1961).

[5] Abramowitz, M., and Stegun, I., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series, (US Government Printing Office, Washington, DC 1964). □□